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**Question Paper Code : 31521**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all branches)

(Regulation 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Solve  $(D^2 - 4)y = 1$ .
2. Convert  $(3x^2D^2 + 5xD + 7)y = 2/x \log x$  into an equation with constant coefficients.
3. Define solenoidal vector function. If  $\vec{V} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + 2\lambda z)\vec{k}$  is solenoidal, find the value of  $\lambda$ .
4. State Green's theorem.
5. Find the constants  $a, b$  if  $f(z) = x + 2ay + i(3x + by)$  is analytic.
6. Find the critical points of the transformation  $w = 1 + \frac{2}{z}$ .
7. Evaluate  $\int_C \frac{z+4}{z^2+2z}$  where  $C$  is the circle  $\left|z - \frac{1}{2}\right| = \frac{1}{3}$ .
8. Find the residue of  $f(z) = \frac{1 - e^{-z}}{z^3}$  at  $z = 0$ .
9. Find the Laplace transform of  $f(t) = \frac{1 - e^{-t}}{t}$ .
10. Find the Laplace transform of the function  $f(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$ .

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 8x e^x \sin x$ . (8)
- (ii) Solve by the method of variation of parameters (8)  
 $2\frac{d^2y}{dx^2} + 8y = \tan 2x$ .
- Or
- (b) (i) Solve  $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + y = e^{\log x}$ . (8)
- (ii) Solve  $\frac{dx}{dt} + 4x + 3y = t$ ;  $\frac{dy}{dt} + 2x + 5y = e^{2t}$ . (8)

12. (a) (i) Show that the vector field  $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$  is irrotational. Find its scalar potential. (6)
- (ii) Verify Stoke's theorem for  $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$  taken around the rectangle formed by the lines  $x = -a$ ,  $x = +a$ ,  $y = 0$  and  $y = b$ . (10)
- Or
- (b) (i) Find  $a$  and  $b$  so that the surfaces.  $ax^3 - by^2z - (a+3)x^2 = 0$  and  $4x^2y - z^3 - 11 = 0$  cut orthogonally at the point  $(2, -1, -3)$ . (6)
- (ii) Verify Gauss Divergence theorem for  $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ , where  $S$  is the surface of the cube formed by the planes  $x = 0$ ,  $x = 1$ ,  $y = 0$ ,  $y = 1$ ,  $z = 0$  and  $z = 1$ . (10)
13. (a) (i) Prove that  $u = e^{-2xy} \sin(x^2 - y^2)$  is harmonic. Find the corresponding analytic function and the imaginary part. (8)
- (ii) Find the bilinear map which maps the points  $z = 0, -1, i$  onto the points  $w = i, 0, \infty$ . Also find the image of the unit circle of the  $z$  plane. (8)
- Or
- (b) (i) Prove that  $w = \frac{z}{1-z}$  maps the upper half of the  $z$ -plane to the upper half of the  $w$ -plane and also find the image of the unit circle of the  $z$  plane. (8)
- (ii) Find the analytic function  $f(z) = u + iv$  where  $v = 3r^2 \sin 2\theta - 2r \sin \theta$ . Verify that  $u$  is a harmonic function. (8)
14. (a) (i) Find the residues of  $f(z) = \frac{z^2}{(z+2)(z-1)^2}$  at its isolated singularities using Laurent's series expansion. (8)
- (ii) Evaluate  $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$ , using contour integration. (8)
- Or
- (b) (i) Show that  $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx = \frac{5\pi}{2}$ . (8)
- (ii) Evaluate  $\int_C \frac{z+1}{(z^2 + 2z + 4)^2} dz$ , where  $C$  is the circle  $|z+1+i|=2$ , by Cauchy's integral formula. (8)
15. (a) (i) Evaluate  $L^{-1} \left( \frac{3s^2 + 16s + 26}{s(s^2 + 4s + 13)} \right)$ . (8)
- (ii) Find the inverse Laplace transform of the following :  $\log \left( \frac{s+1}{s-1} \right)$ . (8)
- Or
- (b) (i) Find  $L^{-1} \left[ \frac{s}{(s^2 + a^2)^2} \right]$  and find  $L^{-1} \left[ \frac{1}{(s^2 + a^2)^2} \right]$  and hence find  $L^{-1} \left( \frac{1}{(s^2 + 9s + 13)^2} \right)$ . (3 + 3 + 2)
- (ii) Using Laplace transforms, solve  $y'' + y' = t^2 + 2t$ ,  $y(0) = 4$  and  $y'(0) = -2$ . (8)